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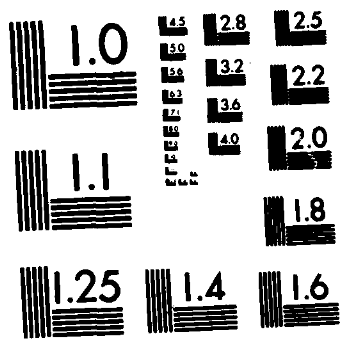

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ACCEPTANCE CONTROL CHARTS
BASED ON THE EXACT AND NORMAL APPROXIMATIONS TO
THE BINOMIAL DISTRIBUTION

Research Report No. 82-6

by

Carlos Amado
Richard S. Leavenworth
Richard L. Scheaffer

RESEARCH REPORT

Industrial & Systems
Engineering Department
University of Florida
Gainesville, FL. 32611

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ACCEPTANCE CONTROL CHARTS BASED ON THE EXACT AND NORMAL APPROXIMATIONS TO THE BINOMIAL DISTRIBUTION

INTRODUCTION

The control chart for fraction rejected, or p-chart, probably is the most widely used of all control chart procedures. It may be applied to one or more than one quality characteristic whether measured on a go, not-go, basis or as variables measurements. So long as the result of an inspection is to classify an item as meeting specifications (acceptable) or not meeting specifications (unacceptable), a single p-chart may be used.

When the sample subgroup size is constant, the chart for $\underline{np}$ may be used conveniently since it records the actual count of rejected units in a subgroup of size $\underline{n}$ rather than the proportion rejected. In either case, the binomial probability density function may be used to model the process.

The standard Shewhart control chart for $\underline{np}$ places the upper and lower control limits at:

$$n\bar{p} \pm 3\sqrt{n\bar{p}(1-\bar{p})}$$

where:

$\underline{n\bar{p}}$ is the average count of rejected units for a series of subgroups of constant size, $\underline{n}$.

$\sqrt{n\bar{p}(1-\bar{p})}$ is the standard deviation of the binomial count ($\sigma_{\underline{np}}$).

Thus these limits are standard 3-sigma limits of the Shewhart control chart. While variation beyond these limits at random should occur very rarely indeed, the same probabilities often associated with random variation beyond 3 sigma limits on an $\bar{X}$ chart should not be applied. Nevertheless, exact probabilities

calculated from the binomial distribution may be found for $\underline{np}$ control charts with 3-sigma limits or any other set of limits which are a multiple of $\sigma_{\underline{np}}$.

In the case of the Acceptance Control Chart, at each point a sample is selected a decision is to be made either to accept the hypothesis that the process is operating as specified, at the acceptable quality level or better, or that the actual quality level is beyond an acceptable level (a higher value of $\underline{p}$). The decision criterion is the upper control limit on np, $\underline{np} + \underline{Z}\gamma\ \sigma_{\underline{np}}$, where γ is the probability of acceptance of the hypothesis with respect to $\underline{p}$ using a normal approximation to the binomial distribution. ($\underline{Z}_\gamma$ equals 3 for the standard Shewhart control chart.)

This paper explores the development of Acceptance Control Charts for binomial counts of the number of units rejected. Two parameters are to be found, one the value of the control limit, and the second, the appropriate (constant) sample size, $\underline{n}$.

The problem is formulated in three ways, using the exact binomial distribution, using a standard normal approximation to the binomial, and using a normalized arcsin transformation. Analytical results of application of the three methods are compared and the results of a simulation study using computer-generated synthetic data are presented.

PROBLEM FORMULATION: EXACT BINOMIAL

If samples are being drawn from a continuous process generating nonconforming items at a constant rate, $\underline{p}$, the binomial distribution describes this process accurately. The probability density function is

$$f(r|n,p) = \binom{n}{r} p^r (1-p)^{n-r} \tag{1}$$

where r = no. of units rejected

n = sample (or subgroup) size.

The probability that the number of units rejected is less than or equal to some fixed value, c, is:

$$P[r < c|n,p] = \sum_{r=0}^{c} \binom{n}{r} p^r(1-p)^{n-r} = \gamma \tag{2}$$

Limits for acceptance control charts, like acceptance numbers for sampling plans, usually are designed by selecting two points on an Operating Characteristic (OC) curve, as illustrated in Figure 1. One point assures that an Acceptable Process Level (APL), p_1, has a high probability of being accepted of at least $1-\alpha$; the other assures that a Rejectable Process Level (RPL), p_2, will have a suitable low probability of acceptance of at most β. α and β are the design risk levels associated with the process quality levels p_1 and p_2, respectively, where p_2 is greater than p_1.

The two equations needed should express the fact that it is desired that the OC curve pass through, or pass as close as possible to, the two points specified as the design criteria, namely, $(p_1, 1-\alpha)$ and (p_2, β). These equations are:

$$P[r < c|n,p_1] = \sum_{r=0}^{c} \binom{n}{r} p_1^r(1-p_1)^{n-r} > 1-\alpha \tag{3}$$

$$P[r < c|n,p_2] = \sum_{r=0}^{c} \binom{n}{r} p_2^r(1-p_2)^{n-r} < \beta \tag{4}$$

They are expressed in inequality form because, for any integer values of n and c, it is unlikely that the cumulative probability can exactly satisfy $1-\alpha$ and β. As the result there are an infinite number of (n,c) pairs satisfying (3) and (4) above some minimum combination. The complete formulation thus requires the further stipulations:

$$\left.\begin{array}{ll} & \text{minimize } n \text{ and } c \\ \text{where:} & p_2 > p_1 \text{ and } 1-\alpha > \beta \end{array}\right\} \tag{5}$$

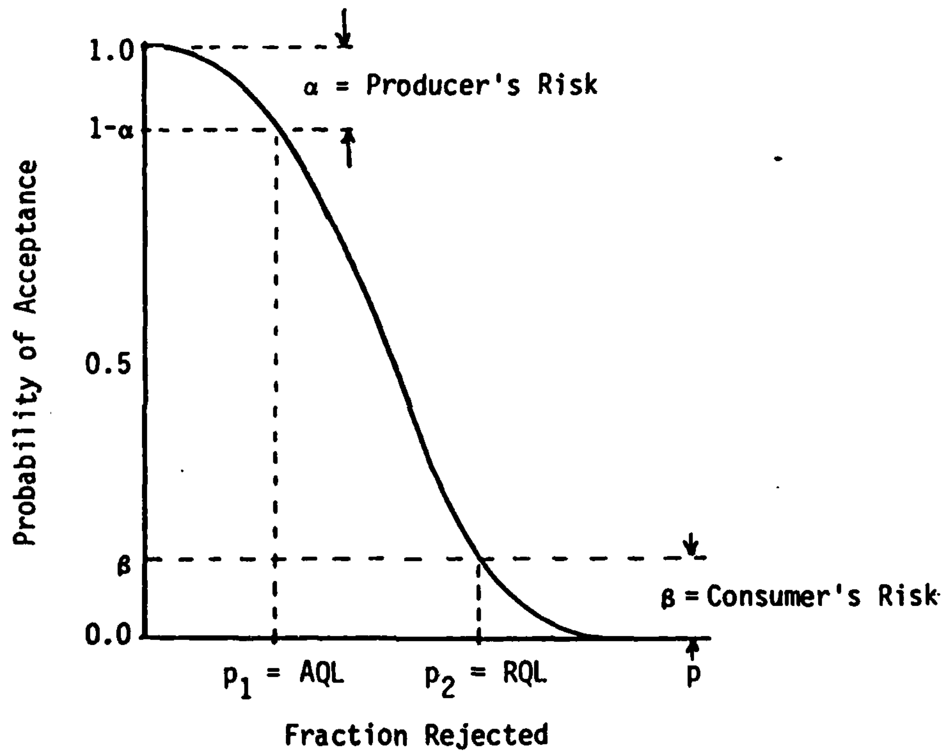

Figure 1. Design Operating Characteristic (OC) Curve for np Chart.

Referring to Fig. 1, equation (3) assures that the resulting OC curve will pass above and to the right of the design curve at the point (p_1, $1-\alpha$). Equation (4) assures that the resulting OC curve will pass below and to the left of the point (p_2,β). Stipulating the choice of the minimum (n,c) pair assures that the resulting OC curve passes as closely as possible to the design curve.

The value of n, of course, yields the constant subgroup size to be used in sampling. Thus the np chart becomes a reasonable and easily understandable alternative to the p chart. The value of c is the maximum count of rejected units that should lead to no corrective action on the process. Only if (c+1) or more units are rejected should action be taken. However, if the Acceptance Control Limit (ACL) is plotted exactly at the value of c, the user may become confused as to whether or not to take action. In accordance with the rules of control chart interpretation, this is not a coin-flip situation. A reasonable procedure is to plot the Acceptance Control Limit at:

$$ACL = c + 0.5 \tag{6}$$

thus avoiding any confusion in chart interpretation.

PROBLEM FORMULATION: NORMAL APPROXIMATIONS

Working with the binomial formula presents a number of mechanical problems some of which are discussed subsequently. Suffice it to say at this point in the discussion that no closed form solution for the values of n and c exists. Its application involves repetitive use of some form of search algorithm. Therefore no one should be surprised that considerable attention and ingenuity have been applied in the development of useful approximations to the cumulative binomial.

Johnson and Kotz (1969) provide a rather extensive survey of binomial approximation techniques and Raff (1956) has compared the accuracy of several of them. The two presented and compared in this study are the standard normal approximation and a normalized arcsin transformation. The standard normal approximation is the most familiar and used as alluded to in the Introduction. It is easy to apply since $\underline{n}$ and $\underline{c}$ may be obtained directly with the use of a slide rule or pocket calculator and a table of the standard normal curve. The arcsin transformation is somewhat more complicated but easily adaptable for use on a programmable pocket calculator.

Standard Normal Approximation

The mean and standard deviation of the binomial distribution are:

$$E(r) = np$$
$$\sigma_r = \sqrt{np(1-p)}.$$

The distribution of the standardized binomial variable

$$Z = (r-np)/\sqrt{np(1-p)}$$

tends to the standard normal distribution as $\underline{n}$ becomes large. (See Johnson and Kotz, 1969.) That is, for any real number, X:

$$\lim_{n\to\infty} P[Z < X] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{X} \exp(-u^2/2)\, du$$

which values may be found in a table of the normal curve or solved for on many programmable pocket calculators. Thus, if given the value of an ACL (say, derived from the exact binomial, non-integer, and equal to $\underline{c} + 0.5$), the probability of $\underline{c}$ or less occurrences with $\underline{n}$ and $\underline{p}$ known, $\Phi(\underline{Z})$, requires only the calculation of:

$$Z_\gamma = (ACL-np)/\sqrt{np(1-p)} \tag{7}$$

and $\Phi(Z_\gamma) = \gamma$ from a cumulative (left-hand) normal curve table.

Arcsin Transformation

The arcsin transformation

$$y = \sin^{-1} \sqrt{(r + 3/8)/(n + 3/4)}$$

produces a random variable, y, which is approximately normally distributed. (See Johnson and Kotz, 1969.) Thus a normalized random variable, Z, produces a statistic, the asymptotic distribution of which is normal, where:

$$Z_\gamma = 2\sqrt{n}\,[\sin^{-1}\sqrt{\frac{c + 3/8}{n + 3/4}} - \sin^{-1}\sqrt{p}] \tag{8}$$

and $\Phi(Z_\gamma) = \gamma$ is then found on a cumulative normal curve table.

PROBLEM SOLUTION - EXACT BINOMIAL

Restating the problem, the objective is to find an (n,c) pair such that:

minimize: n,c

subject to:

$$P[r < c|n,p_1] > 1-\alpha \tag{3}$$

$$P[r < c|n,p_2] < \beta \tag{4}$$

where:

$$p_1 < p_2 \text{ and } \beta < 1-\alpha. \tag{5}$$

Guenther (1969) develops a search procedure for finding (n,c) pairs that satisfy equations 3, 4, and 5*. While Guenther's algorithm is aimed at

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*Actually Guenther's paper is devoted to finding sample sizes (n) and acceptance numbers (c) for single sampling acceptance plans. The procedure, however, is the same. In his paper, the hypergeometric, binomial, and Poisson distributions are used.

deriving plans by hand calculation and the use of tables, it is an iterative, brute-force technique more amenable to computerization than to hand calculation.

Hailey (1980) programmed Guenther's algorithm to find the minimum single sampling acceptance plan satisfying equations (3) and (4). His paper contains the FORTRAN IV computer code for deriving plans using either the binomial distribution or the Poisson.

The algorithm operates basically as follows. For any stipulated value of c, there is a minimal sample size, n, satisfying equation (4). That value is designated n_s, a minimal value of n. For the same value of c, there also exists a maximum value of n satisfying equation (3). That value is designated n_ℓ, a maximum value for n. If the solved value of n_ℓ is less than the solved value of n_s for fixed c, no feasible solution exists for that value of c or any lesser value. If n_ℓ is greater than (or equal to) n_s, then any value of n (a unique solution) is the range $n_s < n < n_\ell$ is feasible for that value of c. In fact, feasible solutions exist for any value of c greater than the designated value, as well. That is, an infinite number of plans exist satisfying equations (3) and (4).

The search procedure begins by setting c equal to zero and solving for n_s and n_ℓ. The value of c is increased by one and the process repeated until a feasible range of n is found. Hailey's program immediately selects the minimum n, n_s, and terminates with a series of output options. A variation of this program was used to derive sample sizes and ACL's based on the binomial distribution. The sample size, n, was set equal to n_s and the Acceptance Control Limit

$$\text{ACL} = c + 0.5$$

in order to avoid any confusion in the intepretation of points falling on the control limit.

PROBLEM SOLUTION: NORMAL APPROXIMATIONS

Hand calculation using the algorithm stated in the previous section would become most tedious and time-consuming. Available tables of the binomial may not cover the ranges of n or p required. To evaluate the binomial where c equal 50 requires the calculation and summing of 51 terms, which is a large task even with the aid of a sophisticated pocket calculator or small computer. This procedure would have to be repeated many times before the minimum (n,c) pair are found. However, approximations usually require the evaluation of only two equations one for n and one for c.

Standard Normal Approximation

If Z_α denotes the value that cuts off an upper tail area of α under the standard normal curve, as illustrated in Figure 2, then the acceptance plan (n,c) pair can be found from the following equations:

$$Z_{1-\alpha} = \frac{c - n\,p_1}{\sqrt{n\,p_1\,(1-p_1)}}$$

$$-Z_{1-\beta} = \frac{c - n\,p_2}{\sqrt{n\,p_2\,(1-p_2)}}$$

Solving these equations simultaneously yields:

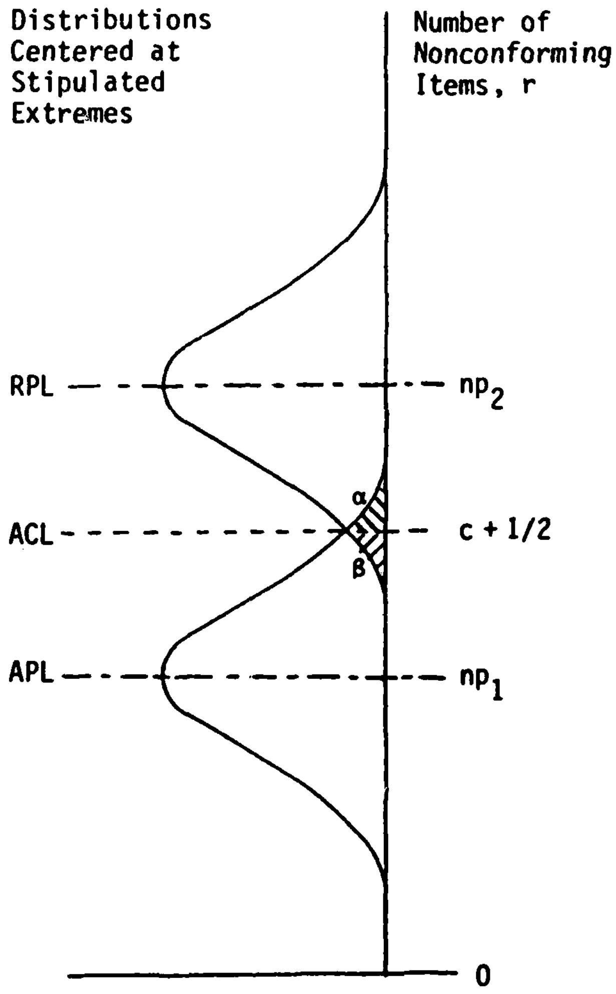

Figure 2. Acceptance Control Charting Scheme for Binomial Counts.

$$n = [(Z_{1-\alpha}\sqrt{p_1(1-p_1)} + Z_{1-\beta}\sqrt{p_2(1-p_2)})/(p_2-p_1)]^2 \tag{9}$$

$$c = Z_{1-\alpha}\sqrt{n\, p_1(1-p_1)} + n\, p_1 \tag{10}$$

$$= -Z_{1-\beta}\sqrt{n\, p_2(1-p_2)} + n\, p_2$$

The values of n and c may be calculated rather quickly by hand or by the use of a programmable pocket calculator. This is the simplest formulation of the problem. The value of n, of course, is rounded to the nearest integer.

Arcsin Transformation

The acceptance control chart plan (n,c) pair can be found by solving the *following equations:*

$$Z_{1-\alpha} = 2\sqrt{n}\,[\sin^{-1}\sqrt{\frac{c+3/8}{n+3/4}} - \sin^{-1}\sqrt{p_1}]$$

$$-Z_{1-\beta} = 2\sqrt{n}\,[\sin^{-1}\sqrt{\frac{c+3/8}{n+3/4}} - \sin^{-1}\sqrt{p_2}]$$

Solving these two equations simultaneously yields:

$$n = \{(Z_{1-\alpha} + Z_{1-\beta})/[2\,(\sin^{-1}\sqrt{p_2} - \sin^{-1}\sqrt{p_1})]\}^2 \tag{11}$$

$$c = (n + 3/4)\,\{\sin\,[Z_{1-\alpha}/(2\sqrt{n}) + \sin^{-1}\sqrt{p_1}]\}^2 - 3/8 \tag{12}$$

Again, the value of n is rounded to the nearest integer. The derived function $\sin^{-1}(x) = \tan^{-1}(x/\sqrt{1-x^2})$ may be used to obtain the inverse sin of x, where $-1 < x < 1$, in computers which do not have an inverse sin function, (Standard Mathematical Tables, 1973).

ANALYSIS

The three methods of calculating (n,c) pairs for Acceptance Control Charts were investigated using the popular risk levels of α equals 0.05 and β equals 0.10. The value of p_1 ranges from 0.005 to 0.06 in increments of 0.005. The value of p_2 changes iteratively in order to maintain specified levels of the discrimination ratio, $D = p_2/p_1$. The value of D ranges from 1.5 to 5.0 in increments of 0.5.

Acceptance Control Chart plans in terms of (n, c) pairs are presented in Table 1. The three methods are labeled BINOMIAL, NORMAL, and ARCSIN to identify, respectively the cumulative binomial distribution, standard normal approximation, and normalized arcsin transformation.

Subgroup sizes, the n's, obtained from the two approximations, ARCSIN and NORMAL, have been rounded to the nearest integer. The acceptance control limits, the c's, where not converted to integers because values from these approximations yield different risk protections depending on whether a continuity correction is added or subtracted from c before it is converted. For example:

Acceptance Control Limit (Integerized)		Truncate Value of	Risk Protection Favored
c'	=	c + 0.5	Type I error
c	=	c	Inconsistent
c*	=	c - 0.5	Type II error

where 0.5 is a continuity correction implicit in Laplace's Theorem. (See Johnson and Kotz, 1969, p.53.) Thus, the conversion of c to integer is performed after determining the continuity correction which will yield the type of risk protection most favorable to the quality control program.

	P1=	0.0050			0.0100			0.0150		
METHOD	D	P2	n	c	P2	n	c	P2	n	c
BINOMIAL	1.5	0.0075	8474.0	53.00	0.0150	4163.0	52.00	0.0225	2774.0	52.00
ARCSINE			8428.0	53.09		4188.0	52.77		2774.0	52.44
NORMAL			8218.0	51.61		4083.0	51.29		2705.0	50.98
BINOMIAL	2.0	0.0100	2473.0	18.00	0.0200	1235.0	18.00	0.0300	822.0	18.00
ARCSINE			2478.0	18.47		1230.0	18.34		814.0	18.21
NORMAL			2373.0	17.52		1178.0	17.39		779.0	17.26
BINOMIAL	2.5	0.0125	1230.0	10.00	0.0250	614.0	10.00	0.0375	409.0	10.00
ARCSINE			1258.0	10.70		623.0	10.62		412.0	10.54
NORMAL			1188.0	9.94		588.0	9.85		389.0	9.77
BINOMIAL	3.0	0.0150	783.0	7.00	0.0300	390.0	7.00	0.0450	260.0	7.00
ARCSINE			792.0	7.52		392.0	7.46		259.0	7.40
NORMAL			739.0	6.85		366.0	6.78		241.0	6.72
BINOMIAL	3.5	0.0175	600.0	6.00	0.0350	299.0	6.00	0.0525	175.0	5.00
ARCSINE			559.0	5.84		276.0	5.79		182.0	5.73
NORMAL			517.0	5.22		255.0	5.16		168.0	5.11
BINOMIAL	4.0	0.0200	462.0	5.00	0.0400	198.0	4.00	0.0600	132.0	4.00
ARCSINE			423.0	4.81		209.0	4.76		138.0	4.71
NORMAL			388.0	4.23		191.0	4.18		126.0	4.13
BINOMIAL	4.5	0.0225	354.0	4.00	0.0450	176.0	4.00	0.0675	117.0	4.00
ARCSINE			336.0	4.11		166.0	4.07		109.0	4.03
NORMAL			306.0	3.56		151.0	3.51		99.0	3.47
BINOMIAL	5.0	0.0250	266.0	3.00	0.0500	132.0	3.00	0.0750	88.0	3.00
ARCSINE			277.0	3.61		136.0	3.58		90.0	3.54
NORMAL			250.0	3.08		123.0	3.04		80.0	3.00

Table 1 (a). Acceptance Control Chart Parameters Obtained by Three Methods.

	P1=	0.0200			0.0250			0.0300		
METHOD	D	P2	n	c	P2	n	c	P2	n	c
BINOMIAL	1.5	0.0300	2043.0	51.00	0.0375	1633.0	51.00	0.0450	1360.0	51.00
ARCSINE			2068.0	52.11		1643.0	51.79		1361.0	51.46
NORMAL			2016.0	50.66		1602.0	50.34		1327.0	50.03
BINOMIAL	2.0	0.0400	616.0	18.00	0.0500	492.0	18.00	0.0600	410.0	18.00
ARCSINE			606.0	18.08		481.0	17.95		398.0	17.82
NORMAL			580.0	17.14		460.0	17.01		380.0	16.88
BINOMIAL	2.5	0.0500	306.0	10.00	0.0625	244.0	10.00	0.0750	203.0	10.00
ARCSINE			306.0	10.45		243.0	10.37		201.0	10.28
NORMAL			289.0	9.69		229.0	9.60		189.0	9.52
BINOMIAL	3.0	0.0600	194.0	7.00	0.0750	155.0	7.00	0.0900	129.0	7.00
ARCSINE			192.0	7.33		152.0	7.27		126.0	7.20
NORMAL			179.0	6.65		141.0	6.59		116.0	6.52
BINOMIAL	3.5	0.0700	131.0	5.00	0.0875	104.0	5.00	0.1050	87.0	5.00
ARCSINE			135.0	5.68		107.0	5.63		88.0	5.57
NORMAL			124.0	5.05		98.0	5.00		81.0	4.94
BINOMIAL	4.0	0.0800	98.0	4.00	0.1000	78.0	4.00	0.1200	65.0	4.00
ARCSINE			102.0	4.67		81.0	4.62		66.0	4.58
NORMAL			93.0	4.08		73.0	4.02		60.0	3.97
BINOMIAL	4.5	0.0900	87.0	4.00	0.1125	70.0	4.00	0.1350	58.0	4.00
ARCSINE			81.0	3.99		64.0	3.95		52.0	3.90
NORMAL			73.0	3.42		57.0	3.37		47.0	3.33
BINOMIAL	5.0	0.1000	65.0	3.00	0.1250	52.0	3.00	0.1500	43.0	3.00
ARCSINE			66.0	3.50		52.0	3.46		43.0	3.42
NORMAL			59.0	2.95		46.0	2.91		38.0	2.86

Table 1 (b).

	P1=	0.0350			0.0400			0.0450		
METHOD	D	p2	n	c	p2	n	c	p2	n	c
BINOMIAL	1.5	0.0525	1165.0	51.00	0.0600	1001.0	50.00	0.0675	889.0	50.00
ARCSINE			1159.0	51.14		1007.0	50.81		890.0	50.48
NORMAL			1130.0	49.71		982.0	49.39		867.0	49.07
BINOMIAL	2.0	0.0700	334.0	17.00	0.0800	292.0	17.00	0.0900	259.0	17.00
ARCSINE			338.0	17.69		294.0	17.56		259.0	17.43
NORMAL			323.0	16.76		281.0	16.63		247.0	16.50
BINOMIAL	2.5	0.0875	174.0	10.00	0.1000	152.0	10.00	0.1125	135.0	10.00
ARCSINE			170.0	10.20		148.0	10.12		130.0	10.03
NORMAL			160.0	9.44		139.0	9.35		122.0	9.27
BINOMIAL	3.0	0.1050	110.0	7.00	0.1200	96.0	7.00	0.1350	85.0	7.00
ARCSINE			107.0	7.14		92.0	7.08		81.0	7.01
NORMAL			99.0	6.46		85.0	6.39		75.0	6.32
BINOMIAL	3.5	0.1225	74.0	5.00	0.1400	65.0	5.00	0.1575	57.0	5.00
ARCSINE			75.0	5.52		65.0	5.47		57.0	5.41
NORMAL			68.0	4.88		59.0	4.83		52.0	4.77
BINOMIAL	4.0	0.1400	56.0	4.00	0.1600	48.0	4.00	0.1800	43.0	4.00
ARCSINE			56.0	4.53		48.0	4.48		43.0	4.44
NORMAL			51.0	3.92		44.0	3.87		38.0	3.82
BINOMIAL	4.5	0.1575	49.0	4.00	0.1800	43.0	4.00	0.2025	38.0	4.00
ARCSINE			44.0	3.86		38.0	3.82		33.0	3.78
NORMAL			39.0	3.28		34.0	3.23		30.0	3.19
BINOMIAL	5.0	0.1750	37.0	3.00	0.2000	32.0	3.00	0.2250	28.0	3.00
ARCSINE			36.0	3.39		31.0	3.35		27.0	3.31
NORMAL			32.0	2.82		27.0	2.77		24.0	2.73

Table 1 (c).

	P1=	0.0500			0.0550			0.0600		
METHOD	D	P2	n	c	P2	n	c	P2	n	c
BINOMIAL	1.5	0.0750	800.0	50.00	0.0825	713.0	49.00	0.0900	654.0	49.00
ARCSINE			795.0	50.16		718.0	49.83		654.0	49.50
NORMAL			775.0	48.76		700.0	48.44		638.0	48.12
BINOMIAL	2.0	0.1000	233.0	17.00	0.1100	212.0	17.00	0.1200	194.0	17.00
ARCSINE			231.0	17.29		209.0	17.16		190.0	17.03
NORMAL			221.0	16.37		199.0	16.25		181.0	16.12
BINOMIAL	2.5	0.1250	121.0	10.00	0.1375	110.0	10.00	0.1500	100.0	10.00
ARCSINE			116.0	9.95		105.0	9.86		95.0	9.78
NORMAL			109.0	9.18		98.0	9.10		89.0	9.01
BINOMIAL	3.0	0.1500	77.0	7.00	0.1650	69.0	7.00	0.1800	63.0	7.00
ARCSINE			72.0	6.95		65.0	6.88		59.0	6.82
NORMAL			67.0	6.26		60.0	6.19		54.0	6.13
BINOMIAL	3.5	0.1750	51.0	5.00	0.1925	46.0	5.00	0.2100	42.0	5.00
ARCSINE			50.0	5.36		45.0	5.31		41.0	5.25
NORMAL			46.0	4.71		41.0	4.66		37.0	4.60
BINOMIAL	4.0	0.2000	38.0	4.00	0.2200	35.0	4.00	0.2400	32.0	4.00
ARCSINE			38.0	4.39		34.0	4.34		31.0	4.30
NORMAL			34.0	3.77		30.0	3.72		27.0	3.67
BINOMIAL	4.5	0.2250	28.0	3.00	0.2475	31.0	4.00	0.2700	23.0	3.00
ARCSINE			30.0	3.74		27.0	3.69		24.0	3.65
NORMAL			26.0	3.14		23.0	3.09		21.0	3.04
BINOMIAL	5.0	0.2500	25.0	3.00	0.2750	23.0	3.00	0.3000	21.0	3.00
ARCSINE			24.0	3.27		22.0	3.23		19.0	3.19
NORMAL			21.0	2.68		19.0	2.64		17.0	2.59

Table 1 (d).

For all three methods, $\underline{n}$ and $\underline{c}$ decrease as the difference between the AQL ($\underline{p_1}$) and RQL ($\underline{p_2}$) increases. This can be observed by either maintaining a constant discrimination ratio ($\underline{D} = \underline{p_2}/\underline{p_1}$) and increasing $\underline{p_1}$, or by maintaining a constant $\underline{p_1}$ and increasing $\underline{D}$.

Tables 2, 3, and 4 indicate the risk over-protection, or under-protection (negative values), which each plan provides. The table values are the differences between actual ($P[k \le c|p]$) values and stipulated ($1-\alpha$ or β) risk protection probabilities. The actual probabilities were obtained using a FORTRAN program, in single precision mode, on a PDP 11/34 computer. The probabilities obtained from this program compared with published binomial probability tables [N.B.S. (1950), Romig (1953), and Harvard (1955)] with an 0.00001 accuracy.

Binomial Distribution

As evidenced in Table 2, ACC plans obtained from the binomial distribution over-protect both the producer and consumer by having low probabilities of committing Type I or Type II errors; i.e., rejecting good quality products or accepting bad quality products. The largest difference between actual and stipulated probabilities is 2.21%. The over-protection is due to the fact that since $\underline{n}$ and $\underline{c}$ are both integers, an OC curve usually cannot pass exactly through points ($\underline{p_1}$, $1-\alpha$) and ($\underline{p_2}$, β), thus the search procedure finds the minimum $\underline{n}$ and $\underline{c}$ which satisfy inequalities (3) and (4).

Standard Normal Approximation

The acceptance control plans obtained from the standard normal approximation generally have smaller subgroup sizes, $\underline{n}$, than the plans obtained from the other two models. As evidenced in Table 4, this method consistantly protects against Type II error when $\underline{c^*}$ is used, but the underprotection

D \ p1	0.0050	0.0100	0.0150	0.0200	0.0250	0.0300	0.0350	0.0400	0.0450	0.0500	0.0550	0.0600
	$R = P[k \leq c \mid p1] - (1-\alpha)$											
1.5	0.0004	0.0008	0.0013	0.0004	0.0001	0.0022	0.0029	0.0006	0.0023	0.0018	0.0005	0.0003
2.0	0.0029	0.0035	0.0048	0.0053	0.0067	0.0069	0.0014	0.0020	0.0035	0.0039	0.0041	0.0052
2.5	0.0011	0.0023	0.0029	0.0039	0.0053	0.0062	0.0065	0.0075	0.0081	0.0095	0.0100	0.0124
3.0	0.0043	0.0055	0.0060	0.0074	0.0081	0.0088	0.0102	0.0112	0.0123	0.0115	0.0147	0.0158
3.5	0.0171	0.0178	0.0004	0.0013	0.0032	0.0029	0.0048	0.0046	0.0075	0.0089	0.0107	0.0118
4.0	0.0199	0.0000	0.0005	0.0027	0.0040	0.0044	0.0040	0.0078	0.0070	0.0103	0.0087	0.0096
4.5	0.0162	0.0172	0.0179	0.0193	0.0190	0.0201	0.0220	0.0221	0.0231	0.0009	0.0242	0.0041
5.0	0.0042	0.0058	0.0062	0.0086	0.0091	0.0105	0.0104	0.0123	0.0146	0.0159	0.0150	0.0159
	$R = \beta - P[k \leq c \mid p2]$											
1.5	0.0001	0.0001	0.0004	0.0003	0.0002	0.0001	0.0002	0.0008	0.0004	0.0012	0.0001	0.0019
2.0	0.0002	0.0004	0.0002	0.0007	0.0005	0.0017	0.0003	0.0008	0.0002	0.0011	0.0031	0.0029
2.5	0.0001	0.0005	0.0014	0.0013	0.0006	0.0011	0.0026	0.0031	0.0041	0.0029	0.0045	0.0006
3.0	0.0005	0.0000	0.0018	0.0007	0.0018	0.0029	0.0018	0.0022	0.0018	0.0075	0.0011	0.0008
3.5	0.0004	0.0006	0.0014	0.0026	0.0009	0.0050	0.0033	0.0074	0.0029	0.0022	0.0005	0.0009
4.0	0.0004	0.0004	0.0028	0.0005	0.0006	0.0031	0.0080	0.0010	0.0073	0.0014	0.0103	0.0118
4.5	0.0009	0.0010	0.0024	0.0012	0.0066	0.0068	0.0030	0.0072	0.0074	0.0037	0.0124	0.0039
5.0	0.0010	0.0008	0.0038	0.0004	0.0037	0.0038	-0.0084	0.0069	0.0037	0.0038	0.0124	0.0144

Table 2. BINOMIAL distribution risk protection difference (R) between actual ($P[k \leq c \mid p]$) and stipulated ($1-\alpha$ or β) probabilities of acceptance plans in Table 1.

p1 / D	0.0050	0.0100	0.0150	0.0200	0.0250	0.0300	0.0350	0.0400	0.0450	0.0500	0.0550	0.0600
	$R = P[k \le c' \mid p1] - (1-\alpha)$;			$c' = (c + 0.5)$ truncated.								
1.5	0.0037	-0.0017	-0.0017	0.0089	-0.0013	0.0033	0.0077	-0.0025	0.0032	0.0081	-0.0022	0.0020
2.0	0.0159	-0.0045	-0.0003	0.0030	0.0063	0.0098	0.0127	0.0149	0.0184	-0.0031	0.0011	0.0045
2.5	0.0095	0.0131	0.0150	0.0173	0.0195	0.0217	-0.0058	-0.0032	0.0009	0.0033	0.0066	0.0093
3.0	0.0153	0.0174	0.0197	0.0215	0.0240	0.0260	-0.0092	-0.0041	-0.0019	0.0003	0.0040	0.0082
3.5	0.0025	0.0055	0.0079	0.0110	0.0133	0.0149	0.0182	0.0198	0.0212	0.0237	0.0263	0.0282
4.0	0.0030	0.0060	0.0080	0.0108	0.0138	0.0160	0.0175	0.0195	0.0231	0.0241	0.0273	0.0293
4.5	0.0303	0.0316	-0.0124	-0.0088	-0.0043	-0.0021	0.0033	0.0043	0.0058	0.0113	0.0150	0.0159
5.0	0.0122	0.0144	0.0174	0.0195	0.0273	0.0234	0.0253	0.0285	0.0288	0.0311	0.0318	0.0336
	$R = \beta - P[k \le c' \mid p2]$											
1.5	-0.0182	-0.0021	-0.0095	-0.0188	-0.0016	-0.0091	-0.0178	-0.0003	-0.0089	-0.0181	0.0004	-0.0066
2.0	-0.0384	0.0007	-0.0049	-0.0100	-0.0166	-0.0235	-0.0304	-0.0349	-0.0453	-0.0008	-0.0077	-0.0133
2.5	-0.0243	-0.0304	-0.0351	-0.0412	-0.0475	-0.0542	0.0012	-0.0019	-0.0088	-0.0122	-0.0184	-0.0236
3.0	-0.0359	-0.0406	-0.0475	-0.0527	-0.0625	-0.0706	0.0055	-0.0029	-0.0051	-0.0074	-0.0144	-0.0238
3.5	-0.0107	-0.0140	-0.0204	-0.0274	-0.0323	-0.0349	-0.0457	-0.0486	-0.0517	-0.0609	-0.0724	-0.0813
4.0	-0.0118	-0.0172	-0.0201	-0.0259	-0.0337	-0.0388	-0.0408	-0.0463	-0.0613	-0.0619	-0.0788	-0.0885
4.5	-0.0806	-0.0842	0.0079	0.0037	-0.0038	-0.0055	-0.0175	-0.0161	-0.0165	-0.0318	-0.0429	-0.0413
5.0	-0.0270	-0.0317	-0.0409	-0.0463	-0.0566	-0.0576	-0.0643	-0.0823	-0.0780	-0.0917	-0.0903	-0.1019
	$R = P[k \le c \mid p1] - (1-\alpha)$;			$c = (c)$ truncated.								
1.5	-0.0106	-0.0017	-0.0176	-0.0060	-0.0013	0.0033	-0.0077	-0.0025	0.0032	-0.0076	-0.0022	0.0020
2.0	-0.0079	-0.0045	-0.0003	0.0030	0.0063	-0.0188	-0.0144	-0.0112	-0.0059	-0.0031	0.0011	0.0045
2.5	-0.0300	-0.0247	-0.0215	-0.0174	-0.0139	-0.0101	-0.0058	-0.0032	0.0009	0.0033	0.0066	0.0093
3.0	-0.0310	-0.0273	-0.0230	-0.0196	-0.0148	-0.0108	-0.0092	-0.0041	-0.0019	0.0003	0.0040	0.0082
3.5	0.0025	0.0055	0.0079	0.0110	-0.0497	-0.0466	-0.0398	-0.0348	-0.0339	-0.0286	-0.0230	-0.0186
4.0	0.0030	0.0060	0.0080	0.0108	0.0138	-0.0557	-0.0528	-0.0484	-0.0402	-0.0381	-0.0303	-0.0255
4.5	-0.0190	-0.0159	-0.0124	-0.0088	-0.0043	-0.0021	0.0033	0.0043	0.0058	0.0113	0.0150	0.0159
5.0	0.0122	0.0144	-0.0692	-0.0644	-0.0576	-0.0550	-0.0504	-0.0418	-0.0413	-0.0349	-0.0334	-0.0282
	$R = \beta - P[k \le c \mid p2]$											
1.5	0.0043	-0.0021	0.0141	0.0064	-0.0016	-0.0091	0.0077	-0.0003	-0.0089	0.0080	0.0004	-0.0066
2.0	0.0054	0.0007	-0.0049	-0.0100	-0.0166	0.0183	0.0134	0.0103	0.0027	-0.0008	-0.0077	-0.0133
2.5	0.0265	0.0224	0.0176	0.0158	0.0117	0.0074	0.0012	-0.0019	-0.0088	-0.0122	-0.0184	-0.0236
3.0	0.0243	0.0237	0.0196	0.0147	0.0108	0.0060	0.0055	-0.0029	-0.0051	-0.0074	-0.0144	-0.0238
3.5	-0.0107	-0.0140	-0.0204	-0.0274	0.0375	0.0364	0.0308	0.0295	0.0283	0.0236	0.0175	0.0130
4.0	-0.0118	-0.0172	-0.0201	-0.0259	-0.0337	0.0399	0.0393	0.0370	0.0297	0.0300	0.0217	0.0172
4.5	0.0145	0.0119	0.0079	0.0037	-0.0038	-0.0055	-0.0175	-0.0141	-0.0165	-0.0318	-0.0429	-0.0413
5.0	-0.0270	-0.0317	0.0447	0.0427	0.0384	0.0386	0.0341	0.0282	0.0312	0.0255	0.0272	0.0226
	$R = P[k \le c^* \mid p1] - (1-\alpha)$;			$c^* = (c - 0.5)$ truncated.								
1.5	-0.0106	-0.0188	-0.0176	-0.0060	-0.0190	-0.0132	-0.0077	-0.0207	-0.0137	-0.0076	-0.0210	-0.0156
2.0	-0.0079	-0.0392	-0.0335	-0.0288	-0.0239	-0.0188	-0.0144	-0.0112	-0.0059	-0.0398	-0.0337	-0.0288
2.5	-0.0300	-0.0247	-0.0215	-0.0176	-0.0139	-0.0101	-0.0605	-0.0564	-0.0499	-0.0460	-0.0406	-0.0361
3.0	-0.0310	-0.0273	-0.0230	-0.0196	-0.0148	-0.0108	-0.0842	-0.0753	-0.0716	-0.0679	-0.0613	-0.0536
3.5	-0.0700	-0.0646	-0.0600	-0.0541	-0.0497	-0.0466	-0.0398	-0.0348	-0.0339	-0.0286	-0.0230	-0.0186
4.0	-0.0818	-0.0761	-0.0722	-0.0666	-0.0603	-0.0557	-0.0528	-0.0484	-0.0402	-0.0381	-0.0303	-0.0255
4.5	-0.0190	-0.0159	-0.1364	-0.1298	-0.1208	-0.1167	-0.1054	-0.1037	-0.1009	-0.0886	-0.0801	-0.0784
5.0	-0.0811	-0.0762	-0.0692	-0.0644	-0.0576	-0.0550	-0.0504	-0.0418	-0.0413	-0.0349	-0.0334	-0.0282
	$R = \beta - P[k \le c^* \mid p2]$											
1.5	0.0043	0.0201	0.0141	0.0064	0.0210	0.0149	0.0077	0.0225	0.0155	0.0080	0.0235	0.0179
2.0	0.0054	0.0333	0.0314	0.0278	0.0232	0.0183	0.0134	0.0103	0.0027	0.0362	0.0315	0.0277
2.5	0.0265	0.0224	0.0176	0.0158	0.0117	0.0074	0.0460	0.0443	0.0402	0.0383	0.0346	0.0316
3.0	0.0243	0.0237	0.0196	0.0167	0.0108	0.0060	0.0552	0.0508	0.0498	0.0488	0.0453	0.0403
3.5	0.0481	0.0455	0.0434	0.0398	0.0375	0.0364	0.0308	0.0295	0.0283	0.0236	0.0175	0.0130
4.0	0.0518	0.0474	0.0483	0.0456	0.0421	0.0399	0.0393	0.0370	0.0297	0.0300	0.0217	0.0172
4.5	0.0145	0.0119	0.0448	0.0453	0.0624	0.0620	0.0572	0.0583	0.0586	0.0524	0.0481	0.0494
5.0	0.0503	0.0486	0.0447	0.0427	0.0384	0.0386	0.0341	0.0282	0.0312	0.0255	0.0272	0.0226

Table 3. Standard NORMAL approximation risk protection difference (R) between actual ($P[k \le c \mid p]$) and stipulated ($1-\alpha$ or β) probabilities of acceptance. Plans in Table 1.

D \ p1	0.0050	0.0100	0.0150	0.0200	0.0250	0.0300	0.0350	0.0400	0.0450	0.0500	0.0550	0.0600
	R= P[k≤c'|p1] − (1−α)			c' = (c +0.5) truncated.								
1.5	0.0077	0.0118	0.0017	0.0067	0.0104	0.0015	0.0059	0.0109	0.0017	0.0060	0.0097	0.0140
2.0	0.0022	0.0050	0.0082	0.0112	0.0135	0.0165	0.0194	0.0210	0.0035	0.0070	0.0092	0.0123
2.5	0.0226	0.0248	0.0264	0.0039	0.0064	0.0087	0.0122	0.0138	0.0168	0.0189	0.0201	0.0229
3.0	0.0299	0.0043	0.0048	0.0095	0.0120	0.0133	0.0154	0.0187	0.0206	0.0228	0.0242	0.0259
3.5	0.0261	0.0278	0.0293	0.0304	0.0318	0.0332	0.0339	0.0046	0.0075	0.0122	0.0142	0.0156
4.0	0.0292	0.0306	0.0317	0.0331	0.0339	0.0357	0.0366	0.0078	0.0070	0.0103	0.0130	0.0142
4.5	0.0220	0.0236	0.0254	0.0266	0.0280	0.0304	0.0314	0.0329	0.0347	0.0344	0.0353	0.0373
5.0	0.0366	0.0378	0.0383	0.0396	0.0091	0.0105	0.0137	0.0160	0.0185	0.0202	0.0198	0.0257
	R= β − P[k≤c'|p2]											
1.5	-0.0089	-0.0163	0.0004	-0.0073	-0.0165	0.0011	-0.0070	-0.0166	0.0019	-0.0075	-0.0162	-0.0253
2.0	0.0019	-0.0030	-0.0083	-0.0139	-0.0199	-0.0257	-0.0338	-0.0387	0.0002	-0.0063	-0.0093	-0.0156
2.5	-0.0396	-0.0462	-0.0517	0.0013	-0.0021	-0.0057	-0.0136	-0.0157	-0.0230	-0.0281	-0.0298	-0.0389
3.0	-0.0610	0.0029	-0.0004	-0.0054	-0.0098	-0.0112	-0.0151	-0.0244	-0.0290	-0.0357	-0.0386	-0.0436
3.5	-0.0422	-0.0484	-0.0537	-0.0592	-0.0635	-0.0709	-0.0727	0.0074	0.0029	-0.0084	-0.0115	-0.0124
4.0	-0.0500	-0.0550	-0.0586	-0.0658	-0.0680	-0.0812	-0.0858	0.0010	0.0073	0.0014	-0.0036	-0.0034
4.5	-0.0249	-0.0284	-0.0338	-0.0359	-0.0398	-0.0517	-0.0564	-0.0615	-0.0735	-0.0626	-0.0655	-0.0830
5.0	-0.0745	-0.0850	-0.0863	-0.0981	0.0037	0.0038	-0.0033	-0.0070	-0.0129	-0.0150	-0.0071	-0.0332
	R= P[k≤c|p1] − (1−α)			c = (c) truncated.								
1.5	0.0077	-0.0021	0.0017	0.0067	-0.0038	0.0015	0.0059	-0.0034	0.0017	0.0060	-0.0051	0.0003
2.0	0.0022	0.0050	0.0082	0.0112	-0.0119	-0.0077	-0.0033	-0.0007	0.0035	0.0070	0.0092	0.0123
2.5	-0.0056	-0.0020	0.0007	0.0039	0.0064	0.0087	0.0122	0.0138	0.0168	-0.0158	-0.0138	-0.0089
3.0	0.0015	0.0043	0.0048	0.0095	0.0120	0.0133	0.0154	0.0187	0.0206	-0.0182	-0.0157	-0.0124
3.5	-0.0142	-0.0107	-0.0077	-0.0048	-0.0024	0.0007	0.0023	0.0044	0.0075	0.0122	0.0142	0.0156
4.0	-0.0132	-0.0102	-0.0078	-0.0045	-0.0026	0.0019	0.0040	0.0078	0.0070	0.0103	0.0130	0.0142
4.5	0.0220	0.0236	0.0254	-0.0296	-0.0264	-0.0205	-0.0173	-0.0143	-0.0094	-0.0108	-0.0084	-0.0026
5.0	-0.0016	0.0014	0.0031	0.0066	0.0091	0.0105	0.0137	0.0160	0.0185	0.0202	0.0198	0.0257
	R= β − P[k≤c|p2]											
1.5	-0.0089	0.0079	0.0004	-0.0073	0.0087	0.0011	-0.0070	0.0086	0.0019	-0.0075	0.0093	0.0019
2.0	0.0019	-0.0030	-0.0083	-0.0139	0.0199	0.0158	0.0099	0.0064	0.0002	-0.0063	-0.0093	-0.0156
2.5	0.0141	0.0097	0.0061	0.0013	-0.0021	-0.0057	-0.0136	-0.0157	-0.0230	0.0261	0.0252	0.0195
3.0	0.0068	0.0029	-0.0004	-0.0054	-0.0098	-0.0112	-0.0151	-0.0246	-0.0290	0.0293	0.0280	0.0253
3.5	0.0240	0.0226	0.0198	0.0168	0.0146	0.0105	0.0099	0.0074	0.0029	-0.0084	-0.0115	-0.0124
4.0	0.0259	0.0234	0.0217	0.0181	0.0173	0.0101	0.0080	0.0010	0.0073	0.0014	-0.0036	-0.0034
4.5	-0.0249	-0.0284	-0.0338	0.0407	0.0392	0.0335	0.0316	0.0296	0.0239	0.0304	0.0296	0.0213
5.0	0.0169	0.0128	0.0128	0.0068	0.0037	0.0038	-0.0033	-0.0070	-0.0129	-0.0150	-0.0071	-0.0332
	R= P[k≤c*|p1] − (1−α)			c* = (c −0.5) truncated.								
1.5	-0.0074	-0.0021	-0.0151	-0.0087	-0.0038	-0.0154	-0.0079	-0.0034	-0.0158	-0.0100	-0.0051	0.0003
2.0	-0.0285	-0.0243	-0.0198	-0.0155	-0.0119	-0.0077	-0.0033	-0.0007	-0.0287	-0.0235	-0.0204	-0.0157
2.5	-0.0056	-0.0020	0.0007	-0.0404	-0.0365	-0.0328	-0.0269	-0.0244	-0.0194	-0.0158	-0.0138	-0.0089
3.0	0.0015	-0.0515	-0.0474	-0.0426	-0.0382	-0.0359	-0.0322	-0.0259	-0.0224	-0.0182	-0.0157	-0.0124
3.5	-0.0142	-0.0107	-0.0077	-0.0048	-0.0024	0.0007	0.0023	-0.0685	-0.0630	-0.0536	-0.0498	-0.0472
4.0	-0.0132	-0.0102	-0.0078	-0.0045	-0.0026	0.0019	0.0040	-0.0749	-0.0770	-0.0704	-0.0647	-0.0625
4.5	-0.0397	-0.0363	-0.0322	-0.0296	-0.0264	-0.0205	-0.0173	-0.0143	-0.0094	-0.0108	-0.0084	-0.0026
5.0	-0.0016	0.0016	0.0031	0.0066	-0.0708	-0.0881	-0.0808	-0.0758	-0.0697	-0.0659	-0.0677	-0.0521
	R= β − P[k≤c*|p2]											
1.5	0.0142	0.0079	0.0201	0.0158	0.0087	0.0231	0.0165	0.0086	0.0242	0.0165	0.0093	0.0019
2.0	0.0353	0.0318	0.0281	0.0242	0.0199	0.0158	0.0099	0.0064	0.0359	0.0314	0.0294	0.0251
2.5	0.0141	0.0097	0.0061	0.0438	0.0418	0.0397	0.0348	0.0337	0.0291	0.0261	0.0252	0.0195
3.0	0.0068	0.0502	0.0484	0.0457	0.0433	0.0428	0.0408	0.0354	0.0330	0.0293	0.0280	0.0253
3.5	0.0240	0.0224	0.0198	0.0168	0.0146	0.0105	0.0099	0.0596	0.0576	0.0521	0.0509	0.0507
4.0	0.0259	0.0234	0.0217	0.0181	0.0173	0.0101	0.0080	0.0604	0.0637	0.0613	0.0594	0.0598
4.5	0.0451	0.0437	0.0413	0.0407	0.0392	0.0335	0.0316	0.0296	0.0239	0.0304	0.0296	0.0213
5.0	0.0169	0.0128	0.0128	0.0068	0.0658	0.0662	0.0637	0.0626	0.0606	0.0602	0.0640	0.0538

Table 4. ARCSINE transformation risk protection difference (R) between actual (P[k≤c|p]) and stipulated (1−α or β) probabilities of acceptance. Plans in Table 1.

against Type I error may be as large as 14%. When $\underline{c'}$ is adopted, protection against Type I error is usually attained, or is at most 1.25% below the stipulated producer's risk probability, but under-protection against Type II error may be as large as 10%. When no continuity correction is used with $\underline{c}$, there is no consistent risk protection, and under-protection against either type of error may be as large as 6%.

Arcsin Transformation

As evidenced in Table 5, acceptance control plans obtained from the arcsin transformation consistently provide one-tail protection. When $\underline{c^*}$ is used, protection against Type II error is attained, but not against Type I error. Under-protection against Type I error may be as large as 9%. When $\underline{c'}$ is adopted, protection against Type I error is attained, but under-protection against Type II error may be as large as 10%. When no continuity correction is used with $\underline{c}$, no consistent protection is attained, and under-protection against either type of error may be as large as 3%.

SIMULATION STUDY

A computer program simulating item manufacture and control charts, written by Davis (1977), was used to analyze the performance of ACC's derived from the binomial distribution, standard normal approximation, and arcsin transformation. Twenty replications of this simulated process were made, using common random numbers for variance reduction.

Process Description

It is desired to have producer and consumer risks of 5% and 10%, respectively. The cost of a Type I error is considered to be greater than that of a Type II error in this simulation; thus, $\underline{c'}$ is used with the

Table 5. BINOMIAL ACC Plan Simulation Results.

SUB GROUP	NONCON-FORMANCES	SUB GROUP	NONCON-FORMANCES	SUB GROUP	NONCON-FORMANCES
1	3	21	3	41	4
2	2	22	2	42	3
3	0	23	2	43	2
4	4	24	4	44	3
5	0	25	3	45	1
6	5	26	3	46	3
7	1	27	3	47	3
8	2	28	2	48	5
9	3	29	3	49	3
10	4	30	2	50	1
11	5	31	3	51	12
12	2	32	5	52	8
13	2	33	0	53	8
14	4	34	1	54	6
15	4	35	1	55	13
16	5	36	1	56	8
17	5	37	3	57	7
18	2	38	2	58	12
19	3	39	2	59	10
20	7	40	0	60	14

approximations because of its increased Type I error protection. In Table 1, when an APL of 1.5% and a discrimination ratio, D, of 3.5 are chosen (i.e., RPL = 5.25%), the acceptance control plans are as follows:

Method	n	c	c'	c*	Plan Chosen
BINOMIAL	175.0	5.00			(175, 5)
ARCSIN	182.0	6.73	7	6	(182, 7)
NORMAL	168.0	5.11	5	4	(168, 5)

Thus the Acceptance Control Limits were positioned at 5.5 for the binomial and standard normal approximation and at 7.5 for the normalized arcsin transformation. The simulation "manufactured" 10,500 items. Initially the manufacturing process had a nonconforming percentage of 1.5, the AQL. After the 8,750th item produced, the process shifted to 5.25% nonconforming, the RQL level. Thus there was a shift from the expected acceptable process level to the rejectable process level to test the plans at the stipulated extremes.

Control Chart Analysis

The control charts in Figures 3 and 4 show the number of nonconforming units found in each subgroup of the three acceptance plans. Tables 5, 6, and 7 contain the simulated data used to plot these Figures. The process shifted to the RPL after the 50th subgroup for the exact binomial plan, and in the middle of the 49th and 53rd subgroups for the arcsin transformation and normal approximation plans, respectively.

The binomial plan made one Type I error (subgroup 20) in 50 subgroups of good quality and no Type II errors were made in the final 10 subgroups. The arcsin transformation plan made one Type I error (subgroup 32) out of 48 good quality subgroups and no Type II errors in the last ten subgroups. The standard normal approximation plan made no Type I errors in the first 52

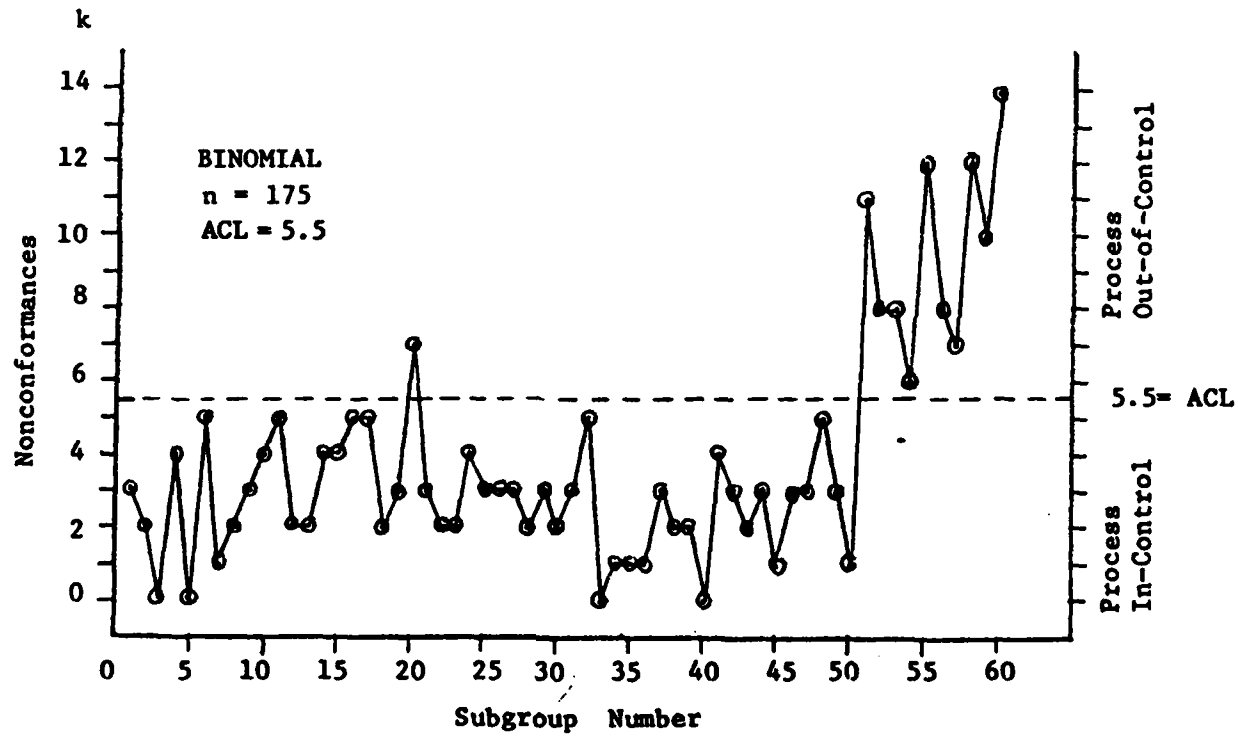

Figure 3. Acceptance Control Chart, Exact Binomial

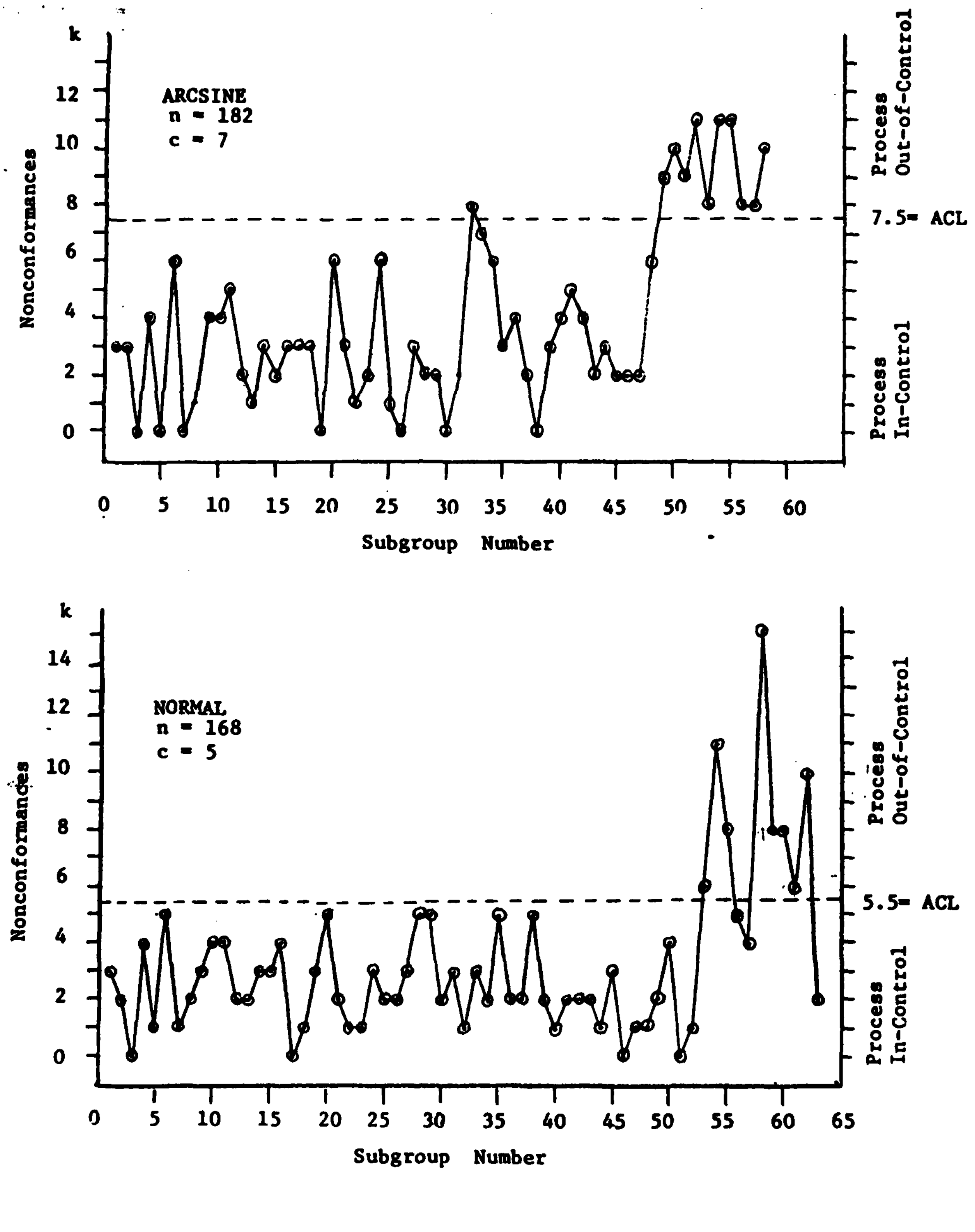

Figure 4. Acceptance Control Charts for ARCSIN and NORMAL approximations.

Table 6. ARCSINE ACC Plan Simulation Results.

SUB GROUP	NONCON-FORMANCES	SUB GROUP	NONCON-FORMANCES	SUB GROUP	NONCON-FORMANCES
1	3	21	3	41	5
2	3	22	1	42	4
3	0	23	2	43	2
4	3	24	6	44	3
5	0	25	1	45	2
6	6	26	0	46	2
7	0	27	3	47	2
8	1	28	2	48	6
9	4	29	2	49	9
10	4	30	0	50	10
11	5	31	2	51	9
12	2	32	8	52	11
13	1	33	7	53	8
14	3	34	6	54	12
15	2	35	3	55	12
16	3	36	4	56	8
17	3	37	2	57	8
18	3	38	0	58	10
19	0	39	3		
20	6	40	4		

Table 7. NORMAL ACC Plan Simulation Results.

SUB GROUP	NONCON-FORMANCES	SUB GROUP	NONCON-FORMANCES	SUB GROUP	NONCON-FORMANCES
1	3	21	2	41	2
2	2	22	1	42	2
3	0	23	1	43	2
4	4	24	3	44	1
5	1	25	2	45	3
6	5	26	2	46	0
7	1	27	3	47	1
8	2	28	5	48	1
9	3	29	5	49	2
10	4	30	2	50	4
11	4	31	3	51	0
12	2	32	1	52	1
13	2	33	3	53	6
14	3	34	2	54	11
15	3	35	5	55	8
16	4	36	2	56	15
17	0	37	2	57	5
18	1	38	5	58	4
19	3	39	2	59	8
20	5	40	1	60	8
				61	6
				62	10
				63	3

subgroups, but had three Type II errors (subgroups 57, 58 and 60) in the last eleven subgroups, when the process had shifted to the RPL.

The total number rejected in a subgroup may be expected to exceed the ACL for either of two reasons, (1) the existence of assignable causes, or (2) the existence of a quality level which exceeds the APL. In either case, the only clue given by the acceptance control chart as to the cause of lack of control is the time at which lack of control at the desired level was observed. For this reason, immediate corrective action should be taken whenever a point exceeds the ACL. This simulation did not include such corrective action because it was desired to observe the consistency of each control chart in providing the desired risk protections.

CONCLUSIONS

Three methods for obtaining subgroup sizes and acceptance control limits were compared. In addition to utilizing the exact binomial distribution, the standard normal approximation and a normalized arcsin transformation were used. Acceptance plans obtained by using the binomial distribution provide the stipulated risk protections (guaranteed over protection) for both producer and consumer. This results from the strict application of the inequality constraints of equations (3) and (4). When a continuity correction of 0.5 is added to (subtracted from) the $\underline{c}$ derived from either of the latter two approximation methods, strict risk protection against Type I (Type II) error is attained, but not against both error types. If no continuity correction is used, the risk protection from these approximations is inconsistent; i.e., risk protection alternates against both types of errors, with no discernable pattern.

For the risk levels studied ($1-\alpha = 0.95$ and $\beta = 0.10$) and the wide range of values of $\underline{p_1}$ and $\underline{D}$ studied, the normalized arcsin transformation yielded results closer to design than did the standard normal approximation. Risk protection losses ranged as high as 2.96% for the producer ($1-\alpha$) and 3.38% for the consumer (β) with no continuity correction factor applied. These are the maximum (underlined) negative numbers in Table 4. With a continuity correction factor of 0.5 added to the solving value of $\underline{c}$, the producer received at least the required protection but the consumer loss of protection reached as high as 9.81% (nearly doubled). With 0.5 subtracted from the solving value of $\underline{c}$, consumer protection at the specified level was assured but loss of producer protection increased to 9.08%, i.e., from a design level of 0.05 to as high as 0.1408. Unless protection at one level is vital, as opposed to protection at the other level, no continuity correction is recommended when the arcsin transformation is to be used.

Results from the standard normal approximation were not as good, in general, as those achieved by applying the arcsin transformation. Without adjustment by a continuity correction factor, loss of producer protection ranged as high as 6.92% and loss of consumer protection as high as 4.29% (negative underlined values in Table 3). With 0.5 added to the solving value of $\underline{c}$, loss of producer protection was reduced to 1.2% but loss of consumer protection was increased markedly to 10.19%. With 0.5 subtracted from the solving value of $\underline{c}$, loss of producer protection increased to 13.64% but consumer protection at the design level was assured. As was the case with the normalized arcsin transformation, no continuity correction can be recommended unless it is imperative to meet (or nearly meet) the design level of protection at <u>either</u> the producer <u>or</u> the consumer quality protection level.

Both the standard normal and arcsin transformation require the evaluation of only two equations to obtain an ACC plan (n,c) pair. These equations easily may be evaluated using Standard Mathematical Tables or a scientific calculator, since they only require the use of square root, sine, and inverse sine functions. The binomial distribution requires at least c^2 evaluations of equation 2, which contains factorial and exponential terms, and a search for the minimum n among various (n,c) pairs that satisfy the inequalities in equations 3 and 4. However, even most home computers have the capability of performing these evaluation and search tasks quickly; they would be tedious and time consuming if performed using tables and/or pocket calculators.

Finally, to assure the desired protection for both producer and consumer, the exact binomial should be used to obtain ACC plans, provided computer facilities are available. The normalized arcsin transformation is preferable to the standard normal approximation because its likely degree of under-protection is about half that of the standard normal. Possible under-protection afforded by the standard normal is about double that of the arcsin in absolute terms.

A complete listing of the computer programs used to develop and evaluate the various plans is provided in the Appendix.

REFERENCES

Davis, Dan T. (1977), "Formulation and Development of a Process Control Simulation Program (SIMCOP)", Dept. of Industrial & Systems Engineering, University of Florida, Gainsville, FL.

Freund, R. A. (1957), "Acceptance Control Charts", Industrial Quality Control, Vol. 14, No. 4, pp. 13-22.

Grant, E. L. and Leavenworth , R. S. (1980), Statistical Quality Control, 5th Ed., McGraw-Hill Book Co., New York, NY.

Guenther, William C. (1969), "Use of the Binomial, Hypergeometric, and Poisson Tables to Obtain Sampling Plans", Journal of Quality Technology, Vol. 1, No. 2, April, pp. 105-109.

Hailey, William A. (1980), "Minimum Sample Size Single Sampling Plans: A Computerized Approach", Journal of Quality Technology, Vol. 12, No. 4, October, pp. 230-235.

Hald, A. (1967), "The Determination of Single Sample Attribute Plans With Given Producer's and Consumer's Risk", Technometrics, Vol. 9, No. 3, August, pp. 401-415.

Harvard University Computation Laboratory (1955), Tables of the Cumulative Binomial Probability Distribution, Harvard University Press, Cambridge, Mass.

Johnson, M. L. and Kotz, S. (1969), Distributions in Statistics -- Discrete Distributions, Ch. 3, Houghton-Mifflin, New York, NY.

Mhatre, S., Scheaffer, R. L. and Leavenworth, R. S. (1981), "Acceptance Control Charts Based on Normal Approximations to the Poisson Distribution", Journal of Quality Technology, Vol. 13, No. 4, pp. 221-227.

National Bureau of Standards (1950), Applied Mathematics Series 6, Tables of the Binomial Probability Distribution, U.S. Printing Office, Washington, D.C., January.

Raff, Morton S. (1956), "On Approximating the Point Binomial", Journal of the American Statistical Association, No. 51, June, pp. 293-303.

Robertson, William H. (1960), Tables of the Binomial Distribution Function for Small Values of P, Sandia Corp. Monograph, SCR-143 TID-4500, 15th Ed., Physics & Mathematics, Alburquerque, NM.

Romig, Harry G. (1953), 50-100 Binomial Tables, John Wiley & Sons, Inc., New York, NY.

Standard Mathematical Tables (1973), 22nd Ed., "Trigonometric Functions in Terms of One Another", CRC Press, Inc., Cleveland, OH.

APPENDIX

```
10 REM*****************************************************************
20 REM*  PROGRAM:  BINOMIAL ACCEPTANCE CONTROL PLAN (N,C)              *
30 REM*****************************************************************
40 DIM SUMLOG(1000)
50 NMAX = 1
60 SUMLOG(NMAX) = 0
70 INPUT "ENTER PRODUCER & CONSUMER'S RISKS ", A,B
80 INPUT "ENTER ACCEPTABLE & REJECTABLE PROCESS LEVELS ", P1,P2
90 GOSUB 130
100  PRINT "SUBGROUP SIZE N = ";N
110  PRINT "CONTROL LIMIT C = ";C
120 END
130 REM*****************************************************************
140 REM*  SUBROUTINE:  SINGLE SAMPLING PLAN SEARCH                     *
150 REM*                                                               *
160 REM*  REFERENCE:   GUENTHER (1969) & HAILEY (1980)                 *
170 REM*                                                               *
180 REM*  GIVEN:  A  =   PRODUCER'S RISK                               *
190 REM*          B  =   CONSUMER'S RISK                               *
200 REM*          P1 =   ACCEPTABLE QUALITY LEVEL (AQL)                *
210 REM*          P2 =   REJECTABLE QUALITY LEVEL (RQL)                *
220 REM*                                                               *
230 REM*  FINDS:  N  =   MINIMUM SUBGROUP SIZE                         *
240 REM*          C  =   NONCONFORMANCE CONTROL LIMIT                  *
250 REM*****************************************************************
260 N =  1
270 C = -1
280 C = C +1
290 P = P2
300 N = N +1
310 GOSUB 370              \ REM***  CALL BINOMIAL (P2, N, C, PACC)
320   IF PACC > B THEN 300
330   P = P1
340 GOSUB 370              \ REM***  CALL BINOMIAL (P1, N, C, PACC)
350   IF PACC < (1-A) THEN 280
360 RETURN
370 REM*****************************************************************
380 REM*  SUBROUTINE:  CUMULATIVE BINOMIAL PROBABILITY                 *
390 REM*                                                               *
400 REM*  GIVEN:  N  =   SUBGROUP SIZE                                 *
410 REM*          C  =   NONCONFORMANCE CONTROL LIMIT                  *
420 REM*          P  =   PROBABILITY OF NONCONFORMANCE                 *
430 REM*                                                               *
440 REM*  FINDS:  PACC =   PROBABILITY OF ACCEPTANCE (CUM. BINOMIAL)   *
450 REM*****************************************************************
460 Q = 1-P
470 CUMULA = Q^N
480 IF C = 0 THEN 600
490 IF N <= NMAX THEN 540   \ REM***  LOG SUMS ALREADY IN MEMORY
500 FOR K = (NMAX+1) TO N   \ REM***  COMPUTE ONLY NEW LOG SUMS
510     SUMLOG(K) = LOG10(K) +SUMLOG(K-1)
520 NEXT K
530 NMAX = N               \ REM***  LARGEST LOG SUM (FACTORIAL) IN MEMORY
540 PLOG = LOG10(P)
550 QLOG = LOG10(Q)
560 FOR K = 1 TO C         \ REM***  COMPUTE CUMULATIVE PROBABILTIY
570     FACTOR = SUMLOG(N) -SUMLOG(N-K) -SUMLOG(K)
580     CUMULA = 10^(FACTOR +K*PLOG +(N-K)*QLOG) +CUMULA
590 NEXT K
600 PACC = CUMULA
610 RETURN
```

```
SINGLE,SINGLE/LI:1=SINGLE

      CC---------------------------------------------------------------------
      CC     PROGRAM:  COMPARES SINGLE SAMPLING PLANS FOR A PROCESS
      CC               WHICH HAS BINOMIALLY DISTRIBUTED DEFECTIVES.
      CC
      CC     PROGRAMMER:  CARLOS AMADO                              FALL '81
      CC
      CC     RESEARCH FOR: DR. R.L. LEAVENWORTH, ISE DEPT, UNIV. OF FLORIDA
      CC---------------------------------------------------------------------
            DIMENSION CLU(2), CLL(2)
            CALL ASSIGN (2,'SINGLE.DAT') ! OUTPUT FILE
        130 CALL PUTCHR (26,5) ! CLEAR CRT SCREEN
            WRITE (5,131)
        131 FORMAT ('0ENTER DATA  (START IN 1ST LETTER OF TITLE)'//
           I           ' PO       ALFA     P1       BETA     ITER8')
            READ (5,133,ERR=130) PO, ALFA, P1, BETA, ETER8
        133 FORMAT (5(F7.4,1X))
            IF (PO.LE.0) GO TO 199
            IF (PO.EQ.P1) GO TO 127
            IF (ETER8.NE.0) GO TO 126
            GO TO 129
      C
        127 WRITE (5,*)
            WRITE (5,*) 'PO CAN NOT EQUAL P1 (ZERO DIVIDE ERROR)'
            GO TO 194
      C
        126 CALL ITER8 (ALFA, PO, BETA, P1, A, ETER8)
            IF (A.EQ.'Y') GO TO 196
            IF (A.EQ.'R') GO TO 194
            GO TO 130
      C
        129 WRITE (2,128) PO, ALFA, P1, BETA
        128 FORMAT (////' GIVEN (PO, A, P1, B):',4(F7.4,2X))
            CALL ARCSIN (ALFA, PO, BETA, P1, SN, C, CLU, CLL)
            WRITE (5,*) SN, C, ' FROM ARCSINE'
            WRITE (2,*) ' '
            WRITE (2,*) 'ARCSINE TRANSFORMATION'
            M= 1
            GO TO 134
      C
        132 CALL NORMBI (ALFA, PO, BETA, P1, SN, C, CLU, CLL)
            WRITE (2,*) ' '
            WRITE (2,*) 'NORMAL APPROXIMATION'
            M= 2
            GO TO 134
      C
        136 CALL GUNTHR (ALFA, PO, BETA, P1, SN, C)
            WRITE (2,*) ' '
            WRITE (2,*) 'EXACT BINOMIAL'
            M= 3
      C
        134 IF (C.EQ.999.) GO TO 199
            CALL EXACT (SN,PO,C,BXLEC,N)
            CALL EXACT (SN,P1,C,P,N)
      C
```

```
          WRITE (2,135) SN, BXLEC, C, P
     135  FORMAT (' SINGLE SAMPLING PLAN:'//
    I           ' SAMPLE SIZE  =',F9.2,5X,
    I           ' P > 1-A =',F9.4/
    I           ' MAX DEFECTS  =',F9.2,5X,
    I           ' P < B    =',F9.4)
          GO TO (132,136,130), M
     194  WRITE (5,*)
          WRITE (5,*)' INPUT ERROR!  TYPE "R" TO RESTART'
     196  READ (5,197) A
     197  FORMAT (A1)
          IF (A.EQ.'R'.OR.A.EQ.'N') GO TO 130
     199  WRITE (5,*)
          STOP '     HAVE A GOOD LIFE'
          END
C--------------------------------------------------------------------
```

SINGLE,SINGLE/LI:1=SINGLE

```
      SUBROUTINE GUNTHR (ALFA, PO, BETA, P1, SN, C)
CC-----------------------------------------------------------------------
CC    PROGRAM:  OBTAINS EXACT BINOMIAL SINGLE SAMPLING PLAN
CC
CC    REF:  GUENTHER, W.C.,"USE OF BINOMIAL, HYPERGEOMETRIC, AND
CC             POISSON TABLES TO OBTAIN SAMPLING PLANS", JOURNAL OF
CC             QUALITY TECH, VOL 1, NO. 2, APRIL 1969, PP 105-109.
CC-----------------------------------------------------------------------
  1   C= -1. ! START AT C=0
      SNS= 1. ! START AT SNS=2
C
  2   C= C+ 1.
C
  3   SNS= SNS+ 1.
      CALL EXACT (SNS, P1, C, BXLEC, N)
      IF (BXLEC.GT.BETA) GO TO 3
C
      SNL= SNS- 1. ! SMALLER SNL NOT NEEDED
  4   SNL= SNL+ 1.
C
      WRITE (5,*) SNS, C, SNL, BXLEC
      CALL EXACT (SNL, PO, C, BXLEC, N)
      IF (BXLEC.GE.(1.-ALFA)) GO TO 4
      SNL= SNL- 1. ! PREVIOUS SNL IS THE ONE WANTED
      IF (SNS.GT.SNL) GO TO 2
      SN= SNS
      RETURN
      END
C-----------------------------------------------------------------------
```

```
SINGLE,SINGLE/LI:1=SINGLE

            SUBROUTINE EXACT (SN, P, C, BXLEC, N)
      CC-----------------------------------------------------------------
      CC    PROGRAM:  COMPUTES EXACT CUMM. BINOMIAL PROBABILITY OF X.LE.C
      CC
      CC    METHOD:
      CC          A. "N" NUMBER OF LOG(10) SUMS ARE COMPUTED ONCE, ONLY,
      CC             AND STORED IN "SUMLOG(I)" VECTOR.
      CC          B. 10**( SUMLOG(I) ) REPRESENTS I-FACTORIAL; THEREFORE,
      CC             FACTORIAL MULTIPLICATIONS ARE REDUCED TO SUMMATIONS,
      CC             ALL FACTORIALS NEEDED ARE IN STORAGE UP TO SUMLOG(N)
      CC          C. WHENEVER ITERATIONS ARE RUN, THE SUMLOG(I)'S IN MEMOR
      CC             ARE NOT RECOMPUTED.
      CC          D. N > C > 0; N-FAC > C-FAC; AND SUMLOG(N) > SUMLOG(C).
      CC          E. SUMLOG(I) SUBTRACTION = I-FACTORIAL DIVISION.
      CC          F. LOG(10) ALLOWS EFFICIENT HANDLING OF LARGE NUMBERS.
      CC
      CC    PROGRAMMER:  CARLOS H. AMADO, ISE DEPT., UNIV. OF FLORIDA
      CC-----------------------------------------------------------------
      CC    GIVEN:     SN      = SAMPLE NUMBER (SIZE)
      CC               P       = PROB. OF DEFECTIVES
      CC               C       = # OF DEFECTIVES IN SAMPLE
      CC
      CC    COMPUTES:  BXLEC   = BINOMIAL PROB (X.LE.C)
      CC               SUMLOG(I) = VECTOR CONTAINING SUM OF LOG(1) THRU LOG
      CC-----------------------------------------------------------------
            VIRTUAL SUMLOG(9000)
            Q = 1.-P
            NN = SN
      C
      C     >>> BINOMIAL PROB. WHEN C=0 <<<
            CSUMS = Q **NN
            IF (C.EQ.0.) GO TO 333
      C
      C     >>> AVOID RECOMPUTING SUMLOG(I)'S ALREADY IN MEMORY <<<
      C
            IF (NN.GT.9000) GO TO 998
            IF (N-NN) 100,211,211
      100   M= N+ 1
      C
      C     >>> COMPUTE N SUMLOGS --EQUIVALLENT TO N-FACTORIAL <<<
      C
            IF (M.GT.1) GO TO 110
            SUMLOG(1) = 0.
            M = 2
       110  DO 111 I= M, NN
                  SUMLOG(I) = ALOG10(FLOAT(I)) +SUMLOG(I-1)
       111  CONTINUE
      C
      C     >>> COMPUTE C CSUMS --EQUIVALLENT TO SUM OF PROB COMBINATIONS
      C         I.E., CUMMULATIVE BINOMIAL DISTRIBUTION COMPUTATION
      C
       211  KC = C
            N= NN
      C
```

```
SINGLE,SINGLE/LI:1=SINGLE

      C     >>> SUM (ACCUMULATE) PROBABILITIES <<<
      C
            PLOG= ALOG10(P)
            QLOG= ALOG10(Q)
            DO 322 K= 1, KC
                  CSUMS = 10.**( SUMLOG(N) -SUMLOG(N-K) -SUMLOG(K)
           Z                         +K*PLOG +(N-K)*QLOG ) +CSUMS
       322  CONTINUE
      C
       333  BXLEC= CSUMS
            RETURN
       998  WRITE (5,999) SN, P, C
       999  FORMAT ('OWHOOPS!!!   SAMPLE SIZE =',F10.2//
           Z        ' AT P =',F10.4,'  AND C =', F10.4)
            STOP
            END
      C-----------------------------------------------------------------------
```

```
SINGLE,SINGLE/LI:1=SINGLE

      SUBROUTINE NORMBI (ALFA, PO, BETA, P1, SN, C, CLU, CLL)
CC-----------------------------------------------------------------------
CC    PROGRAM:  COMPUTES A SINGLE SAMPLING PLAN USING THE
CC              NORMAL APPROXIMATION FOR A BINOMIAL DISTRIBUTION
CC
CC-----------------------------------------------------------------------
      DIMENSION CLU(2), CLL(2), ZV(12), ZT(12), Z(2)
      DATA ZV(1)/2.326/, ZV(2)/2.054/, ZV(3)/1.881/, ZV(4)/1.751/,
     I     ZV(5)/1.645/, ZV(6)/1.555/, ZV(7)/1.474/, ZV(8)/1.405/,
     I     ZV(9)/1.34/, ZV(10)/1.282/, ZV(11)/1.036/, ZV(12)/.842/
      DO 29 I= 1, 10
 29       ZT(I)= 1.- I/100.
      ZT(11)= .85
      ZT(12)= .80
      PZ= 1.-ALFA
      K= 1
 30   DO 31 I= 1, 12
          IF (PZ.NE.ZT(I)) GO TO 31
          Z(K)= ZV(I)
          GO TO (32,33), K
 31   CONTINUE
      WRITE (5,*) 'ERROR: ALFA OR BETA NOT IN DATA STATEMENT'
      C= 999.
      RETURN
C
 32   PZ= 1.-BETA
      K= 2
      GO TO 30
C
 33   PQO= (1- PO)* PO
      PQ1= (1- P1)* P1
      SN= ((Z(1)* SQRT(PQO) + Z(2)* SQRT(PQ1))/ (P1- PO))**2
      C= Z(1)* SQRT(SN*PQO)+ SN* PO
C
      RETURN
      END
C-----------------------------------------------------------------------
```

SINGLE,SINGLE/LI:1=SINGLE

```
      SUBROUTINE ARCSIN (ALFA, PO, BETA, P1, SN, C)
CC------------------------------------------------------------------------
CC    PROGRAM:  SINGLE SAMPLING PLAN USING AN ARCSINE NORMALIZING
CC              TRANSFORMATION* TO APPROXIMATE A BINOMIAL PROCESS.
CC
CC              1.  NORMAL                        ARCSINE TRANSFORMATION
CC              2.  FREEMAN & TUCKEY (F & T)         "            "
CC
CC    REF:  JOHNSON, N, S. KOTZ,'DIST. IN STATISTICS -DISCRETE DIST.'
CC              HOUGHTON MIFFLIN CO, BOSTON, 1969, P 65.
CC
CC    GIVEN:    ALFA  = PRODUCER'S RISK PROBABILITY OF REJECTION
CC              BETA  = CONSUMER'S  "         "       "  ACCEPTANCE
CC              PO    = ACCEPTABLE PROCESS LEVEL
CC              P1    = REJECTABLE    "      "
CC
CC    COMPUTES: SN    = SAMPLE SIZE
CC              C     = MAX NUMBER OF DEFECTIVES IN ACCEPTANCE
CC
CC    VARIABLES:
CC              Z(1)    = Z(1-ALFA)
CC              Z(2)    = Z(1-BETA)
CC              SINV(1) = ARCSINE OF SQRT(1-ALFA)
CC              SINV(2) =    "     "    "  (1-BETA)
CC              SS(1)   = SAMPLE SIZE OF NORMAL ARCSINE TRANSFORMATIO
CC              SS(2)   =    "     "    "  F & T     "        "
CC              D(1)    = ACCEPTABLE # DEFECTIVES IN NORMAL      "
CC              D(2)    = ACCEPTABLE # DEFECTIVES IN F & T       "
CC------------------------------------------------------------------------
      DIMENSION Z(2), SINV(2), SS(2), D(2), ZT(12), ZV(12), P(2)
      DATA ZV(1)/2.326/, ZV(2)/2.054/, ZV(3)/1.881/, ZV(4)/1.751/,
     I     ZV(5)/1.645/, ZV(6)/1.555/, ZV(7)/1.474/, ZV(8)/1.405/,
     I     ZV(9)/1.34/, ZV(10)/1.282/, ZV(11)/1.036/, ZV(12)/.842/
      DO 29 I= 1, 10
 29        ZT(I)= 1.- I/100.
      ZT(11)= .85
      ZT(12)= .80
      P(1)= PO
      P(2)= P1
C
      PZ= 1.- ALFA
      K= 1
 30   DO 31 I= 1, 12
           IF (PZ.NE.ZT(I)) GO TO 31
           Z(K)= ZV(I)
           SQRP= SQRT(P(K))
           SINV(K)= ATAN( SQRP/ SQRT(-SQRP*SQRP+1) ) ! ARCSIN
           GO TO (32,33), K
 31   CONTINUE
      WRITE (5,*) 'ERROR: ALFA OR BETA NOT IN DATA STATEMENT'
      C= 999.
      RETURN
C
 32   PZ= 1.-BETA
```

```
SINGLE,SINGLE/LI:1=SINGLE

            K= 2
            GO TO 30
      C
       33   SS(2)= ( (Z(1)+Z(2))/ (SINV(2)-SINV(1)) )**2
            SS(1)= SS(2) /4
            X= SINV(1)+ Z(1)/ (2*SQRT(SS(1)))
            D(1)= (SS(1)+0.75) * (SIN(X)**2) -(3./8.)
      C
            SN= SS(1)
            C= D(1)
            RETURN
            END
      C-----------------------------------------------------------------
```

```
SINGLE,SINGLE/LI:1=SINGLE

      SUBROUTINE ITER8 (ALFA, PO, BETA, P1, A, ETER8)
CC--------------------------------------------------------------------
CC    PROGRAM:  DETERMINES SEVERAL SINGLE SAMPLING PLANS
CC              BY CHANGING PARAMETER VALUES IN EACH ITERATION.
CC--------------------------------------------------------------------
      DIMENSION SS(3), CC(3), BO(3), B1(3)
      COMMON /ETER/ AL,AS,AH,BL,BS,BH,POL,POS,POH,P1L,P1S,P1H
      WRITE (5,101)
 101  FORMAT ('OENTER ITERATION (STEP) AND HIGHEST VALUES'//
     Z       ' (ALFA) ALFA    (PO)    PO      (P1)    P1      (BETA) BETA'
      READ (5,103) AS, AH, POS, POH, P1S, P1H, BS, BH
 103  FORMAT (8F7.4)
C
      IF (AS.LT.0.OR.AH.LT.ALFA) GO TO 194
      IF (BS.GT.0) GO TO 104
      BS= AS
      BH= AH
C
 104  IF (POS.LT.0.OR.POH.LT.PO.OR.P1S.LT.0.OR.P1H.LT.P1.OR.
     Z    P1S.LT.POS.OR.BH.LT.BETA) GO TO 194
      AL= ALFA
      BL= BETA
      POL= PO
      P1L= P1
      IF (ETER8.EQ.8.) GO TO 114
      GO TO 110   ! BEGIN ITERATIONS
C
 106  P1= P1+ P1S
      IF (P1.LE.P1H) GO TO 110
      P1= P1L
      PO= PO+ POS
      IF (PO.LE.POH) GO TO 110
      PO= POL
      ALFA= ALFA+ AS
      IF (ALFA.LE.AH) GO TO 110
      ALFA= AL
      BETA= BETA+ BS
      IF (BETA.LE.BH) GO TO 110
      WRITE (5,*)
      WRITE (5,*)' END OF ITERATIONS!  END SESSION?  [Y/N]'
      A= 'Y'
      RETURN
 194  A='R'
      RETURN
C
 110  CALL ARCSIN (ALFA, PO, BETA, P1, SS(1), CC(1))
      CALL NORMBI (ALFA, PO, BETA, P1, SS(2), CC(2))
      CALL GUNTHR (ALFA, PO, BETA, P1, SS(3), CC(3))
      DO 112 I= 1, 3
            CALL EXACT (SS(I), PO, CC(I), BO(I), N)
 112        CALL EXACT (SS(I), P1, CC(I), B1(I), N)
      A1= 1.-ALFA
      WRITE (2,113) A1, BETA, PO, P1, SS(3), CC(3), BO(3), B1(3),
     Z     SS(1), CC(1), BO(1), B1(1), SS(2), CC(2), BO(2), B1(2)
```

```
 113  FORMAT ('0GIVEN (1-A, B, P0, P1):',4(F7.4,','))//
     Z          ' SINGLE SAMPLING PLAN:      N',7X,'C      P>1-A    P<B'/
     Z          ' EXACT BINOMIAL',8X,2F8.2,2F8.4/
     Z          ' ARCSINE TRANSFORMATION',2F8.2,2F8.4/
     Z          ' NORMAL APPROXIMATION  ',2F8.2,2F8.4)
      GO TO 106
 114  CALL ITER88 (ALFA, P0, BETA, P1, A)
      RETURN
      END
C-------------------------------------------------------------------------
```

SINGLE,SINGLE/LI:1=SINGLE

```
      SUBROUTINE ITER88 (ALFA, PO, BETA, P1, A)
C===========================================================================
C     ROUTINE:  COMPARES DESCRIMINATION RATIOS (P1/PO)
C===========================================================================
      DIMENSION SS(9), CC(9), BO(9), B1(9), PA(3), PB(3)
      COMMON /ETER/ AL,AS,AH,BL,BS,BH,POL,POS,POH,P1L,P1S,P1H
 112  WRITE (5,*)
      WRITE (5,*)'ENTER D-STEP FOR P1'
      READ (5,*) DS
      IF (DS.LT.0) GO TO 112
C
 114  A1= 1.-ALFA
      WRITE (2,115) A1,ALFA,BETA
      WRITE (1,115) A1,ALFA,BETA
 115  FORMAT ('OGIVEN (1-A, A, B):',3(F7.4,','))
      PA(1)= POL
      PB(1)= P1L
      PH= P1H+ 0.001
      SP1= P1S
C
 116  D= SP1/POS
      DO 111 M= 1, 3
            DD= PB(M)/PA(M)
            IF (DD+0.01.LE.D.OR.DD-0.01.GE.D) GO TO 128
            IF (PB(M).GT.PH) GO TO 126
C
            CALL GUNTHR (ALFA, PA(M), BETA, PB(M), SS(M), CC(M))
            CALL ARCSIN (ALFA, PA(M), BETA, PB(M), SS(M+3), CC(M+3))
            CALL NORMBI (ALFA, PA(M), BETA, PB(M), SS(M+6), CC(M+6))
C
            IF (M.GE.3) GO TO 111
            PA(M+1)= PA(M)+ POS
            PB(M+1)= PB(M)+ SP1
 111  CONTINUE
C
      DO 122 M= 1, 9
            I= SS(M)+ 0.5
 122        SS(M)= I
 117  DO 121 M= 1, 3
            DO 119 I= M, 9, 3
               CALL EXACT (SS(I),PA(M),CC(I),BO(I),N)
 119           CALL EXACT (SS(I),PB(M),CC(I),B1(I),N)
 121  CONTINUE
      WRITE (2,123) (PA(I),I=1,3),D,PB(1),SS(1),CC(1),PB(2),SS(2),CC(
     H                          PB(3),SS(3),CC(3),D,PB(1),SS(4),CC(
     I                          PB(2),SS(5),CC(5),PB(3),SS(6),CC(6)
     T                          D,PB(1),SS(7),CC(7),PB(2),SS(8),CC(
     S                          PB(3),SS(9),CC(9)
 123  FORMAT ('OMETHOD   D\P1=',3(F8.4,'      N',7X,'C  ')/
     B        ' BINOMIAL',F6.1,3(F8.4,F8.1,F8.2)/
     A        ' ARCSINE ',F6.1,3(F8.4,F8.1,F8.2)/
     R        ' NORMAL  ',F6.1,3(F8.4,F8.1,F8.2))
      WRITE (1,125) (PA(I),I=1,3),D,PB(1),BO(1),B1(1),PB(2),BO(2),B1(
     P                          PB(3),BO(3),B1(3),D,PB(1),BO(4),B1(
```

```
     E                                    PB(2),BO(5),B1(5),PB(3),BO(6),B1(6
     T                                    D,PB(1),BO(7),B1(7),PB(2),BO(8),B1(
     S                                    PB(3),BO(9),B1(9)
 125  FORMAT ('OMETHOD    D\P1=',3(F8.4'   P>1-A    P<B    ')/
     M          ' BINOMIAL',F7.1,9F8.4/' ARCSINE ',F7.1,9F8.4/
     E          ' NORMAL  ',F7.1,9F8.4)
      PB(1)= PB(1)+ DS
      SP1= (D+ 0.5) *PA(2) -PB(1)
      GO TO 116
C
 126  BETA= BETA+ BS
      IF (BETA.LE.BH) GO TO 114
      BETA= BL
      ALFA= ALFA+ AS
      IF (ALFA.LE.AS) GO TO 114
      WRITE (5,*)
      WRITE (5,*)'END OF ITERATIONS!  END? [Y/N]'
      A='Y'
      RETURN
C
 128  WRITE (5,*)
      WRITE (5,*)'DESCRIMINATION RATIO ERROR; D=',D,'  DD=',DD
      A= 'R'
      RETURN
      END
C-----------------------------------------------------------------------
```

```
SINGLE,SINGLE/LI:1=SINGLE

0001          SUBROUTINE NORMAL (X, U, SD, ZTAB, TABZ)
        CC-----------------------------------------------------------------
        CC     PROGRAM:  NORMAL PROBABILITY DISTRIBUTION APPROXIMATION
        CC
        CC               CUM. NORMAL= 1 - F(X) * (B1 * T + B2 * T**2 + B3 * T*
        CC                                        + B4 * T**4 + B5 * T*
        CC               MIN. ACCURACY: +- 0.000 000 075
        CC
        CC     REF:  ABRAMOWITZ, M., I. STEGUN, EDS. 'HANDBOOK OF MATH.
        CC               FUNCTS. WITH FORMULAS, GRAPHS, AND MATH TABLES', APPL
        CC               MATH SERIES #55, WASH. DC, NAT'L BUREAU OF STDS., 196
        CC
        CC     GIVEN:    X       = SAMPLE STATISTIC
        CC               U       = DISTRIBUTION MEAN
        CC               SD      = STANDARD DEVIATION
        CC
        CC     COMPUTES: ZTAB    = NORMAL CUMMULATIVE PROBABILITY
        CC               TABZ    = (1 - ZTAB)
        CC-----------------------------------------------------------------
0002          DATA B1/0.319381153/, B2/-0.356563782/, B3/1.781477937/, DUMY/0.
            Z     B4/-1.821255978/, B5/1.330274429/, CONST/0.39894228/
0003          Z= (X - U)/SD
0004          IF (Z.GE.0.) GO TO 21
0006          Z = -Z
0007          DUMY = 1.
0008     21   T = 1./(1.+Z *0.2316419)
        C     C = 1/SQRT(2* 3.141526536); SEE DATA
0009          F = C* EXP( -(Z**2)/2.)
0010          TABZ = F* (B1* T +B2* T**2 +B3* T**3 +B4* T**4 +B5* T**5)
0011          IF (DUMY.EQ.0.) GO TO 23
0013          TABZ = 1.-TABZ
0014     23   ZTAB = 1.-TABZ
0015          RETURN
0016          END
```

SINGLE,SINGLE/LI:1=SINGLE

```
            SUBROUTINE REVNOR (X, U, SD, ZTAB, TABZ)
      CC-----------------------------------------------------------------
      CC    PROGRAM:  REVERSE NORMAL PROBABILTY DISTRIBUTION
      CC
      CC                X= T -(C0+ C1*T+ C2* T**2)/(1+ D1*T+ D2* T**2+ D3* T*
      CC
      CC                MIN. ACCURACY:  +- 0.000 45
      CC
      CC    REF:  HAISTINGS, CECIL JR, 'APPROXS FOR DIGITAL COMPS', PRINCET
      CC                N.J, PRINCETON UNIV. PRESS, 1955.
      CC
      CC    GIVEN:  U      = DISTRIBUTION MEAN
      CC            SD     = STANDARD DEVIATION
      CC            ZTAB   = NORMAL CUMMULATIVE PROBABILITY
      CC            TABZ   = (1 - ZTAB)
      CC
      CC    COMPUTES: X    = POINT STATISTIC
      CC-----------------------------------------------------------------
            DATA C0/2.515517/, C1/0.802853/, C2/0.010328/, DUMY/1./,
           I     D1/1.432788/, D2/0.189269/, D3/0.001308/
            F= ZTAB
            TABZ= 1- ZTAB
            IF (ZTAB.GE.1) GO TO 35
            IF (ZTAB.LT.0.5) GO TO 31
            DUMY= 0.
            F= 1.- ZTAB
       31   TT= ALOG (1./F**2)
            T= SQRT (TT)
            XN= C0+ C1* T+ C2* T**2
            XD= 1.+ D1* T+ D2* T**2 +D3* T**3
            X= T- XN/ XD
            IF (DUMY.EQ.0.) GO TO 33
            X= -X
       33   X= X* SD+ U
            GO TO 37
       35   X= 999.999
       37   RETURN
            END
      C------------------------------------------------------------------
```

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END

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